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Mathematics and the Myth of Neutrality

> Suppose we loosely define a religion as any discipline whose foundations rest on an element of faith, irrespective of any element of reason which may be present. Quantum mechanics for example would be a religion under this definition. But mathematics would hold the unique position of being the only branch of theology possessing a rigorous demonstration of the fact that it should be so classified.
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> F. De Sua cited in H. Eves *Mathematical Circles* (Boston: Prindle, Weber and Schmidt, 1969).

> Mathematics and the sciences have no business with the divine religion
>
> Aharon ben Zalman Emmerich Gumpertz (1723-1769)
> From "Ma'amar hamada ' in Megale sod (Hamburg,1765).[1]

What do Christian beliefs have to do with mathematics? Surely mathematics is neutral: doesn't two and two make four no matter what beliefs someone has? If mathematics were neutral then it would be independent of beliefs or philosophies. However, this is far from the case. In discussing various philosophies of mathematics Paul Ernest makes this comment:

> Mathematical truth ultimately depends on an irreducible set of assumptions, which are adopted without demonstration. But to qualify as true knowledge, the assumptions require a warrant for their assertion. There is no valid warrant for mathematical knowledge other than demonstration or proof. Therefore the assumptions are beliefs, not knowledge, and remain open to doubt.[2]

Mathematics thus rests on belief. This echoes an observation of cosmologist John Barrow:

> One would normally define a 'religion' as a system of ideas that contains statements that cannot be logically or observationally demonstrated. Rather, it rests either wholly or partially upon some articles of faith. Such a definition has the amusing consequence of including all the sciences and systems of thought that we know; Gödel's theorem not only demonstrates that mathematics is a religion, but shows that mathematics is the only religion that can prove itself to be one![3]

In other words: beliefs are integral to mathematics; mathematics cannot be neutral.

The philosophy of mathematics

The philosophy of mathematics can be classified into two main schools: the absolutists and the fallibilists. The absolutist sees mathematics as being objective, fixed, incorrigible and absolutely certain – hence the name: the emphasis is on mathematic (singular). The fallibilist sees it as being corrigible, subjective, incomplete, ever changing and fallible: it is mathematic*s* (plural). An absolutist math teacher may well teach by emphasizing routine mathematical tasks and give the expectation that there is one correct answer. The fallibilist math teacher may well emphasize the collaborative, problem solving, investigational approach to math. How we view math affects how we teach – of course, this is not the only influence upon teaching styles, a math absolutist may well teach differently according the pedagogical beliefs she may hold. Two plus two may equal four, but the issue is whose concept of 'two' and whose concept of 'plus' and 'equals' are we using? In the following we will examine different ways in which philosophies view mathematics.

Absolutist views of mathematics

Absolutism is a blanket term for several distinct views, which can include: logicism, formalism, intuitionism, Platonism and empiricism. The one thing they all have in common is that at heart all mathematics rests upon a firm foundation. They all, however, violently disagree over what is that foundation.

Logicism

Advocates of *logicism* include: G.W. Leibniz (1646-1716), Gottolb Frege (1848-1925), Bertrand Russell (1872-1970), Alfred North Whitehead (1861-1947) and Rudolf Carnap (1891-1970). Carnap stated baldly their article of faith:

> Logicism is the thesis that mathematics is reducible to logic, hence nothing but a part of logic.[4]

Here logic becomes absolutized. However, the foundation of logic was soon shown to be sinking sand. It proved to be over complicated, obscure and ambiguous.[5] The paradoxes of set theory also added to its demise. As Russell later confessed:

> Having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of very arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable.[6]

Formalism

David Hilbert (1862–1943), Johann von Neumann (1903–1957) and Haskell B. Curry (1900–1982) are the main figures associated with *formalism*. Formalism is so-named because its adherents see mathematics as a formal language. This school was dominant in the mid twentieth century.

It was Kurt Gödel who undermined the whole formalist programme in 1931. He 'proved' that there would always be certain true statements that can never be proved. Typical of such a statement is:

This statement cannot be proved true.

If it can be proved true then the statement is false, if it cannot be proved true then it is true!

For the formalist mathematical objects do not exist, mathematics is reduced to formulae, to linguistics.

Intuitionism

Immanuel Kant (1724–1804) and Leopold Kronecker (1823–1891) are the forebears of mathematical intuitionism, though best known are L. E. J. Brouwer (1881–1966) and his student Arend Heyting (1898–1980). As Heyting has asserted:

> The intuitionist mathematician proposes to do mathematics as a natural function of his intellect, as a free, vital activity of thought. For him, mathematics is a product of the human mind[7]

Likewise Brouwer:

> Mathematical entities do not have existence in our conception of nature anymore than nature itself.[8]

Mathematics, the constructivists contend, is invented not discovered: it is a human construction; nevertheless it is absolutist in the sense that it sees the foundations of math as being certain. The intuitionists sought to base mathematics upon what was self-evident. The enterprise failed when they couldn't agree upon what was self-evident: what was 'self-evident' apparently wasn't self-evident! They rejected anything that wasn't intuitive. This meant they rejected a large amount of math, which made the approach untenable to most mathematicians. Typical was Kronecker's rejection of irrational numbers.

As mathematics, for the intuitionists, is constructed from intuitively obvious ideas, it takes place primarily in the mind. Intuition is the foundation for mathematics. Hence, Kronecker's famous quote: 'God made the integers, but all else is the work of man'.

Platonism

Cosmologist John Barrow aptly describes the platonic view of mathematics: ' "Pi" really is in the sky.'[9] Mathematical objects and structures have a real objective existence 'out there somewhere'; hence mathematics is discovered not invented. As the name suggests, this views origins is in the thought of Plato. In this sense Pythagoras and Leibniz were Platonists. More recent adherents include Georg Cantor (1845–1918), Gilbert Hardy (1877–1947) and Kurt Gödel (1906–1978), together with the physicists Heinrich Hertz, Richard Feynman, John Barrow, Roger Penrose and Paul Davies. Many Christians, such as John Polkinghorne, have also adopted a form of mathematical Platonism.

Platonism is remarkable in that it is so successful; most practicing physicists and mathematicians have adopted it. It is however, inherently religious: it attributes the attributes of divinity to an external eternal realm that contains mathematical ideas. As Clouser notes: '... by regarding this hypothetical realm as having independent existence [Platonists] accord it the status of divinity!'[10]

Empiricism

For the empiricist, math is based on empirical generalizations, sense data. This was the position of the utilitarian philosopher John Stuart Mill (1806–1873). There are at least two valid objections to empiricism. When our experience contradicts math we don't reject mathematical truths and much of math is abstract and does not have its origins in the world of experience.

Of absolutism and fallibilism, it is absolutism that could provide any justification for a neutral view of mathematics. However, as we have seen all absolutist views rely upon some aspect of creation to provide a foundation. They all have a faith commitment in some aspect of reality as being self-existent, and this is hardly a neutral position! Moreover, the consensus now is that absolutist views of mathematics are untenable and unreliable. Imre Lakatos (1922–1974), perhaps the most trenchant critic of absolutist approaches to math, has shown that the quest for certainty in the above theories leads to a vicious circle: in the quest for certainty they all rely upon unprovable assumptions. Neither elephants nor tortoises were stable enough a foundation! In the next section we fallibilist views of mathematics are examined, we will see that fallibilist views actually support the notion that mathematics is not neutral.

Fallibilist views of mathematics

Conventionalism

For the *conventionalist* the foundations of mathematics rest on linguistic conventions. Conventionalists include: the moderates, for example, W. V. O. Quine (1908–2000) and C. Hempel (1905–1997); and the non-absolutists, for example Wittgenstein (1889–1951). Though not strictly fallibilist, it can be accommodated within a fallibilist view. As one philosopher, Machover, points out it has often been the refuge of defeated logicists.

Conventionalism points out the social structure of mathematical knowledge and little else. It reduces mathematics to the lingual and social aspects of reality.

Social constructivism

Social constructivism is the philosophy of mathematics that Ernest proposes. It draws upon conventionalism and Lakatos' quasi-empiricism. Reuben Hersch and Ernst von Glaserseld have advocated similar approaches, labeled 'humanist' and 'radical constructivism', respectively. Ernest summarizes the social constructivist view:

The grounds for describing mathematical knowledge as a social construction and adopting this name are threefold:

(i) The basis of mathematical knowledge is linguistic, conventions and rules, and language is a social convention.
(ii) Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge.
(iii) Objectivity itself will be understood to be social.

Point (ii) is perhaps the weak point: the shift from subjective to objective knowledge in the social constructivist position is by publication. This seems to imply that mathematics rather than being '*out* there somewhere' is '*in* there somewhere'! This causes more problems than it solves. It could be construed to suggest that journal referees are the arbiters of truth; and what happens if conflicting accounts of mathematics are both published are they both deemed to be true?

Point (iii) takes objectivity as social agreement. Objective knowledge can thus be false! This opens the way up to accusations of relativism. Ernest acknowledges these weaknesses but is, to my mind, unsuccessful in defusing them.

Point (i) is taken from conventionalism; consequently it suffers the same weakness as that position: it is a reduction of mathematics to the lingual and social aspects of reality.

One of the main tenets of the fallibilist view of mathematics is that it is a product of human activity. This undermines the neutrality view of maths, as the following argument shows:

1. We all have a worldview
2. A worldview is shaped by religious commitments
3. All human activity is shaped by worldviews
4. Mathematics is a human activity

Therefore,

5. Mathematics is shaped by worldviews that are religious commitments

The conclusion for a fallibilist can only be that mathematics is shaped by one's worldview: hardly a neutral position! The view then that mathematics is neutral is untenable.

In place of a conclusion

The fallibilist view actually supports the idea that mathematics is not neutral. The absolutist view, which at first would appear to support the belief that mathematics is neutral, rests on the belief that maths can be reduced to one or two aspects of creation. Each of these aspects of creation are believed to be self-existent and uncreated, this means that they have divine attributes, hence these beliefs are inherently religious: hardly a neutral position.

None of the philosophies alone can provide a suitable basis for a Christian approach to mathematics. A Christian approach to mathematics may well have to acknowledge with absolutists an ontological objectivity, but also with fallibilists an epistemological subjectivity, but we must recall that mathematics is a creation of God: the only thing uncreated is God. This is the point in which a Christian view of mathematics radically departs from the above-mentioned philosophies. Mathematics does not rest on some aspect of creation; its foundation is God the creator of all things. Christian beliefs should and do shape mathematics.

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[1] Cited in Michael Deakin and Hans Lausch 'The Bible and pi' *The Mathematical Gazette* **82** (no 494) (1998) p. 165.

[2] Paul Ernest *The Philosophy of Mathematics Education* (Falmer, Basingstoke 1991) p. 14.

[3] *The World Within the World* (Oxford University Press, Oxford, 1988) p. 257.

[4] Cited in Paul Benacerraf and Hilary Putnam (ed.) *Philosophy of Mathematics: Selected Readings* (Blackwell, Oxford, 1964) p.31.

[5] Imre Lakatos *Mathematics, Science and Epistemology* (Cambridge University Press, Cambridge, 1978) p. 14

[6] Cited in Ruben Hersh *What is Mathematics, Really?* p. 151.

[7] Cited in Benacerraf and Putnam (ed.) *Philosophy of Mathematics* p.42.

[8] Cited in Benacerraf and Putnam (ed.) *Philosophy of Mathematics* p.67

[9] John Barrow *The World Within the World* p. 241.

[10] R. A. Clouser *The Myth of Religious Neutrality: An Essay on the Hidden Role of Religious Beliefs in Theories* (University of Notre Dame Press, Notre Dame, 1991) p. 123 My debt to Clouser's book goes far beyond this footnote!