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Beliefs Shape Mathematics

An examination of several philosophies of mathematics reveals that mathematics is not neutral but is shaped by beliefs that are at heart religious. If beliefs shape mathematics than it follows that Christian beliefs can shape mathematics. The paper concludes by sketching the contours of one Christian approach to mathematics education.

1. Introduction

What is mathematics? Is it a construction of the human mind, a summary of sensory experience, an explanation of objective reality, a formal language, a social convention, a useful tool, or a means of economic advance? How we answer this question has important and inescapable consequences for mathematics education. Unfortunately, the vast majority of mathematics teachers are unaware of the philosophical presuppositions that underlie mathematics.

Paul Ernest in an important work has shown that there is a relationship between the espoused and enacted beliefs of the mathematics teacher.[1] Beliefs are integral to mathematics. This has profound implications for a Christian approach to mathematics. It exposes the myth of the neutrality of mathematics, thus Christian beliefs can shape mathematics education.

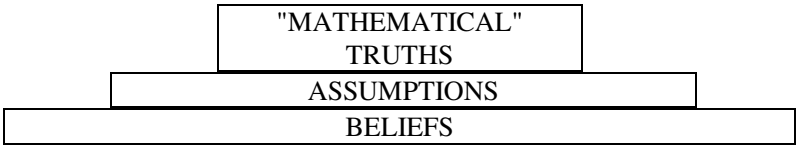
2. The philosophy of mathematics

The philosophy of mathematics can be classified into two main schools: the absolutists and the fallibilists. The former sees mathematics as being absolutely certain; the latter as being corrigible and fallible.

In discussing the various philosophies of mathematics Ernest makes this apposite comment:

> Mathematical truth ultimately depends on an irreducible set of assumptions which are adopted without demonstration. But to qualify as true knowledge, the assumptions require a warrant for their assertion. There is no valid warrant for mathematical knowledge other than demonstration or proof. Therefore the assumptions are beliefs, not knowledge, and remains open to doubt.[2]

[Mathematics thus rests on belief. We can illustrate this schematically:



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This echoes an observation of cosmologist John Barrow:

> One would normally define a "religion" as a system of ideas that contains statements that cannot be logically or observationally demonstrated. Rather, it rests either wholly or partially upon some articles of faith. Such a definition has the amusing consequence of including all the sciences and systems of thought that we know; Godel's theorem not only demonstrates that mathematics is a religion, but shows that mathematics is the only religion that can prove itself to be one![3]

Both absolutist and fallibilist philosophies of mathematics rest on presuppositions that cannot be attested, they all reduce mathematics to certain aspects of reality which are regarded in some sense as self-existent and self-evident. These presuppositions are however, as Barrow puts it, "articles of faith" or as Ernest has it, "beliefs". We will examine some philosophies of mathematics in order to show that this is so..

2.1 Absolutist views of mathematics

Absolutism is a blanket term for several distinct views, which include: logicism, formalism, constructivism (intuitionism), platonism and conventionalism.

Advocates of *logicism* include: G.W. Leibniz (1646-1716), Gottolb Frege (1848-1925), Bertrand Russell (1872-1970), Alfred North Whitehead (1861-1947) and Rudolf Carnap (1891-1970). Carnap baldly stated their article of faith:

> Logicism is the thesis that mathematics is reducible to logic, hence nothing but a part of logic.[4]

However, logic was soon shown to be sinking sand. It proved to be over complicated, obscure and ambiguous.[5] The paradoxes of set theory also added to its demise.

David Hilbert (1862-1943), John von Neumann (1903-1957) and Haskell B. Curry are the main figures associated with *formalism*. Formalism is so-named because its adherents see mathematics as a formal language. This school was dominant in the mid twentieth century.

It was Kurt Godel who undermined the whole formalist programme in 1931. He "proved" that there will always be certain true statements that can never be proved. Typical of such a statement is:

This statement cannot be proved true.

For the formalist mathematical objects do not exist, mathematics is reduced to formulae. For mathematics, reality is reduced to linguistics.

Immanuel Kant (1724-1804) and Leopold Kronecker (1823-1891) are the forebears of *constructivism*, though the best-known constructivists[6] are the intuitionists L. E. J. Brouwer and Arend Heyting. Mathematics, the constructivists contend, is invented not discovered. Consequently, this is a modified subgroup to absolutism: *progressive absolutism.*[7]

Mathematics is constructed from intuitively obvious ideas; it takes place primarily in the mind. Intuition is the foundation for mathematics. Hence, Kronecker's famous quote: "God made the integers, but all else is the work of man".

Barrow aptly describes the platonic view of mathematics: "'Pi' really is in the sky".[8] Mathematical objects and structures have a real objective existence "out there somewhere"; hence mathematics is discovered not invented. As the name suggests this view has its origins in the thought of Plato, more recent adherents include Georg Cantor (1845-1918), Gilbert Hardy (1877-1947) and Kurt Godel (1906-1978), together with the physicists Heinrich Hertz, Richard Feynman, John Barrow, Roger Penrose and Paul Davies.

Platonism is remarkable in that it is so successful in being adopted by most practicing physicists and mathematicians. It is however, inherently religious: it attributes the attributes of divinity to an external eternal realm which contains mathematical ideas.

2.2 Fallibilist views of mathematics

For the *conventionalist* the foundations of mathematics rest on linguistic conventions. Conventionalists include: the moderates, for example, W. V. O. Quine (1908-) and C. Hempel; and the non-absolutists, for example Wittgenstein (1889-1951). Though not strictly fallibilist, it can accommodate a fallibilist view. Machover points out that it has often been the refuge of defeated logicists.

Conventionalism points out the social structure of mathematical knowledge and little else. It reduces mathematics to the lingual and social aspects of reality.

Social constructivism is the philosophy of mathematics that Ernest proposes. It draws upon conventionalism and Lakatos' quasi-empiricism. He writes:

> The grounds for describing mathematical knowledge as a social construction and adopting this name are threefold:
>
> (i) The basis of mathematical knowledge is linguistic, conventions and rules, and language is a social convention.
> (ii) Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge.
> (iii) Objectivity itself will be understood to be social.

Point (ii) is perhaps the weak point: the shift from subjective to objective knowledge in the social constructivist position is by publication. This seems to imply that mathematics rather than being *"out* there somewhere" is "*in* there somewhere"! This causes more problems than it solves. [It could be taken to suggest that journal referees are the arbiters of truth.]

Point (iii) takes objectivity as social agreement. Objective knowledge can thus be false! This opens the way up to accusations of relativism. Ernest acknowledges these weaknesses but is, to my mind, unsuccessful in defusing them.

Point (i) is taken from conventionalism; consequently it suffers the same weakness as that position: it is a reduction of mathematics to the lingual and social aspects of reality.

All the foregoing philosophies of mathematics as we have seen are reductionistic. They reduce reality to one or two aspects, for example : logic, lingual and social and so on; in one case reality is merely a human construction. None of them *alone* provide a suitable basis for a Christian approach to mathematics.

Different philosophical perspectives *about* mathematics result in differences *within* mathematics.[9] This is evident from the Kronecker's rejection of irrationals. These different perspectives are also religious, because they regard an aspect of creation as self-existing.

3. The Philosophy of Mathematics Education

Ernest's book deals with five educational "ideologies" (see below), these provide a useful way to see how ideologies shape mathematics education. His identification of the five ideologies arises out of his use of Perry theory, and the work of Williams[10] and Cosin.[11]

For each approach he attempts to outline their:

Political ideology

Moral values
Theory of society
Epistemology
Theory of the child
Educational aims
Theory of school mathematics
    aims of mathematics educations
    theory of learning mathematics
    theory of resources
    theory of assessment of mathematics learning
    theory of ability in mathematics
    theory of social diversity in mathematics

3.1 Industrial trainers

These are identified with the New Right and exemplified by Thatcherism. They hold to an absolutist view of mathematics. Mathematics is a body of true, unadulterated facts, skills and theories. Mathematics is neutral, therefore social issues are irrelevant to mathematics and mathematics education.

The aim of mathematics education is that pupils become numerate and have a good grasp of the "basics". This is achieved by a drill and rote approach, with emphasis on hard work and self-application. The learner is passive - an empty bucket - and teaching involves merely the impartation of information.

3.2 Technological pragmatist

Again these adhere to an absolutist philosophy of mathematics; it is an unquestioned body of useful information. The recent *Enterprising Mathematics* course is a good example of this outlook. It is utilitarian in outlook; mathematics should be relevant to industrial needs. Hence, the *Enterprising Mathematics* approach is to equip pupils for the enterprising culture. This tradition absolutises the economic aspect of life: mathematics education is to further the economic utility of the future work force.

3.3 Old humanist

For the old humanist mathematics is a body of objective, neutral knowledge; this is a platonic view. Mathematics education is the transmission of this pure, hierarchically structured body of knowledge. The early *Schools Mathematics Project* schemes exemplified this approach. The teacher's role is reduced to that of explainer and lecturer. The emphasis is on "teaching mathematics", as opposed to "teaching children".[12]

3.4 Progressive educator

This tradition, yet again, assumes absolutism, albeit a progressive absolutism. Whereas the old humanists are subject-centred, the progressive educators are experience and child-centred; they adopt a process approach to education, the teachers are facilitators of learning, not transmitters of knowledge. The purpose of mathematics is to contribute to the development and flowering of the individual. To this end, mathematics teaching will involve carefully structured environments for exploration, it will be integrated and interdisciplinary.

3.5 Public educators

This is the approach Ernest favours. In adopting a social constructivist approach to mathematics it is the only one of the five ideologies that rejects absolutism.

Williams identifies the main tenets of this position as:

> all members of society have a natural right to be educated, and that any good society depends on governments accepting this principle as a duty.[13]

"Development of democratic citizenship" is then the aim of mathematics education. The teaching of mathematics has a

number of components to further this aim (p 208-9):

1 genuine discussion, both student-student and student-teacher, since learning is the social construction of meaning;
2 cooperative groupwork, project-work and problem solving, for confidence, engagement and mastery;
3 autonomous project, exploration, problem posing and investigative work, for creativity, student self-direction and engagement through personal relevance;
4 learner questioning of course contents, pedagogy and the modes of assessment used for critical thinking; and
5 socially relevant materials, projects and topics, including race, gender and mathematics, for social engagement and empowerment.

4. Towards a view of Christian Mathematics Education

If Ernest is correct - and I believe he is - that our overall epistemological and ethical perspectives influence our view of the nature of mathematics, or indeed any other curriculum subject, then it has tremendous ramifications for Christians. At least it means that we have to take philosophy seriously! But primarily, it means that a distinctively Christian approach to mathematics is possible! Both these conclusions mean hard work: we have to think through and discern what our current philosophical assumptions are and modify them to become more in line with a Christian perspective. Renewing our minds is a lifetime's occupation; it does not happen automatically at conversion.

Where does this leave us for *a* Christian approach to mathematics education? There is of course no one Christian approach to mathematics.[14] Gene Chase has identified three possible approaches taken by Christians: the applicational, the incarnational and the philosophical.[15] The first sees a Christian approach in the ways that mathematics is used: to be "Christian" it should be used in ways that comport with Christian values. The second sees the integration of mathematics and Christianity in the Christian person: Christian mathematics exists in so far as it is a Christian doing the mathematics. Here the emphasis is on the *context* rather than the *content* of mathematics. The philosophical view sees philosophical assumptions shaping mathematics - an approach that is validated by Ernest. The approaches are not mutually exclusive! The context *and* content of mathematics as well as the mathematician's lifestyle need to be shaped by Christian beliefs.

The following is an extremely tentative and brief attempt to outline what I would see as one approach for Christians.

Biblical teaching is clear: education is the primary responsibility of the parents. Parents of course can opt to delegate that responsibility to a school. Parental involvement should be encouraged in Christian schools.

Christian education is primarily Christocentric. Jesus is the source and sustainer of all things, includes the school, the curriculum and mathematics.

Theory of society: society consists of institutions and organisations. Institutions include: the family, the state, the church/ synagogue/ mosque. Each of these institutions are called to be obedient to certain God-given norms. Each of these institutions are autonomous under God, hence the state should not dictate what the church should do and vice versa. This concept is known as "sphere sovereignty", and was developed by the one time prime minister of Holland Abraham Kuyper. The school is another organisation, the purpose or calling of which is to educate.

Political ideology: the state is the instrument of God to do justice in the political arena. It is the responsibility of Christians to call the state to do just that. Thus, taking part in the political process through voting, lobbying, demonstrations and even taking part in civil disobedience, if the state does not act justly, is part of Christian discipleship. The role of the state as regards education is that it should ensure that it can be done justly; i.e. it should ensure that schools are fulfilling their calling and that they have sufficient resources and funding to do that. The state should not dictate to the schools what should and should not be taught.

Moral value: on a personal level these would include the fruit of the spirit; on a social level we could add shalom, stewardship, social justice, mercy, a love for learning,... .

***Epistemology*:** knowledge is multifaceted. It is based on revelation - both special revelation, i.e. through the Bible and the direction of the Holy Spirit; and general revelation; i.e. that in creation.

Knowledge involves (at least) the knower, the knowable and the knowing process. The emphasis on the knower is stressed

by Polanyi's work[16]: the personal is always involved in knowledge; the objectivity of knowledge is a myth. The knowable is rooted in reality, there is a "metaphysical objectivity". The other aspect is "epistemological subjectivity"; the knowing process is fallible coloured by our worldview. Epistemology is rooted in ontology.

Theory of the child: each child is unique and created in the image of God. There has been much debate as to what this phrase means and there are as many answers as there are theological systems because the nature of humanity is integral to one's worldview. There are only a few verses that mention the image of God. However, the biblical and extra-textual contemporary evidence seems to suggest that it is both a verb and a noun: we image God and we are to be imagers of God. The first implies that it is something we are, the second something we do. The latter arises out of the former.

Included within its meaning is the task to subdue and rule the creation: this is not a mandate for dominion but responsible stewardship. Part of the calling to be human is to steward creation. This means developing and cultivating creation according to God-given norms.

All humans are religious being, we have an innate desire to worship something, be it a football team, popgroup, family, money, science or whatever. The history of humanity is of one allegiance to different idols. In the history of mathematics these idols have included: number, logic, intuition, sense-data etc.

Educational aim: *t*he purpose of education includes the following:

i For each person to fulfil her/his God-given potential.
ii To develop and open up God's good creation as implied in the cultural mandate.
iii To develop a Christian world and life view, which will provide a framework for critiquing, subverting, and thus begin to transform, secular ideologies

Philosophy of mathematics: mathematics is a collective term for a number of related fields: arithmetic, geometry, topology, statistics, probability, and others. All of them investigate and open up the two most basic aspects of reality: the umerical and spatial. (Applied mathematics also deals with a third aspect: the kinematic.) Mathematics is a human activity and therefore comes replete with human limitations: it is fallible, corrigible, culture-laden and value-laden. However, it is based in creation - it is not arbitrary or the product of social agreement. This is where a Christian view of mathematics diverges from the social constructivist view. This Christian view of mathematics does justice to both epistemological subjectivity and ontological objectivity.

Mathematical aims: mathematics is based in created reality. It is not neutral; beliefs shape mathematical theories. Hence, mathematics education should (in no particular order):

i be placed in a historical and cultural context;
ii be rooted in reality;
iii be integrated with other subjects (mathematics deals with the two most basic aspects of reality, the numerical and the spatial, these aspects are basic to all other curriculum subjects. The integration with other subjects, particularly science, reveals the role of mathematics as a tool to help fulfil the cultural mandate);
iv describe the beauty and order of creation, and to help students understand creation.
v reveal the attributes of God (Rom 1:20; Ps 19:1) - particularly the faithfulness of God to his creation exemplified in his laws and the lawfulness of creation; and
vi provide fun and enjoyment.

Theory of teaching and learning: a Christian approach should celebrate the diversity of both creation and student. Each student will have individual needs and learning preferences. The teaching of mathematics should reflect this. It will be pluralistic and eclectic, drawing on the strengths of the diversity of learning theories available, be they constructivist, Piagetian or behaviourist; and the strengths of the recent developments in mathematics education

The teaching room should be rooted in the real world and not in lists of meaningless abstract questions. Discussion will also play an important part.

Where possible the history of mathematics should be included as this shows the interaction between mathematics and culture. Possible strategies could include imaginary dialogues between mathematicians and story telling about mathematicians.[17]

Theory of Resources: as the learning and teaching of mathematics is pluralistic and diverse, so too should be the

resources drawn upon. They will include all diverse aspects of creation. Including: newspaper articles, other school departments, notably science and geography, puzzles and games, individual and group work, projects and mathematics trails, ...

Theory of assessmen: a Christian approach would reject the "bureaucratic syllogism":

> If it's not basic it's not worth teaching
> Basics are assessable
> So anything not assessable is not a basic
> Therefore anything not assessable is not worth teaching[18]

Is there a Christian form of assessment? Unfortunately, very little work has been done on assessment from a Christian perspective.[19] Should we assess? How do we assess? What do we assess? These are three important questions to consider. Assessment like any human activity is value-laden. What we assess indicates what we view as important.

Assessment is part and parcel of the National Curriculum, consequently we *have to* assess. This is one reason why we assess! It is not, nor should be, the *only* reason. As Christians we have a responsibility to think deeper and go beyond pragmatic considerations.

Assessment should be formative and summative. Formative assessment is to provide feedback and to allow the correction of error. This can be used as a means of accounting to parents and as a diagnostic tool. In general formative assessment is preferred to summative assessment, which provides information and feedback to the pupil, teacher, governors and parents in terms of an overall assessment of achievement.

Assessment is, and should be, limited in its scope. We can assess behaviour and the products/ processes that the student produces/ pursues; but we cannot, and indeed should not, assess personhood.

5. Strengths and weaknesses of this Christian Approach

The strength of this Christian approach is that it rests on a Christian worldview; i.e. a worldview that is consistent, coherent and comprehensive.

Its weaknesses are twofold. Firstly, it is strong on critique but weak on implementation of alternatives. This is an indictment not of the Christian worldview but of the Christian's who adhere to such a view - and I include myself in such a view - and have failed so far to develop a comprehensive alternative. One notable exception is the primary mathematics package : *The Shape and Number of Things*.[20]

Is other major weakness is its emphasis on a pluralistic and eclectic approach in its teaching, learning and resources. Potentially this could be a strength: drawing on the best of other curriculum alternatives. The problem lies in the process of discerning those aspects which comport well with a Christian view and rejecting those that do not. The use of a multiplicity of resources may provide students with mixed messages about the nature of mathematics.

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[2] Ernest *The Philosophy of Mathematics*, p. 14.

[3] *The World Within the World* (Oxford: Oxford University Press, 1988) p. 257.

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[6] The mathematical school of constructivism should not be confused with the constructivist approach to education. See S. Bishop and J. Carpenter "Constructivism: an introduction and critique" *Spectrum* vol. 25(2) 1993 pp. 147-58

[7] Ernest *Philosophy of Mathematics Education* p. 28.

[8] John Barrow *The World Within the World* p. 241; compare also his *Theories of Everything* (Oxford: Oxford University Press, 1990) ch. 9

[9] R. A. Clouser *The Myth of Religious Neutrality: An Essay on the Hidden Role of Religious Beliefs in Theories* (Notre Dame: University of Notre Dame Press, 1991) p. 119. My debt to Clouser's book goes far beyond this footnote!

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[18] Mike Bottery *The Morality of the School: The Theory and Practice of Values in Education* (London: Cassell, 1990) p. 27

[19] Though see Brian V. Hill "Is assessment compatible with agape?" *Spectrum* vol 25(1) (1993) 39-56.

[20] Calvin Jongsma and Trudy Baker *The Shape and Number of Things: An Integrated Math Curriculum for the Elementary School* (Toronto: Curriculum Development Center). Also see the Charis project based at Stapleford Education Centre, Nottingham..

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